Unsteady Separation Phenomena in a Two-Dimensional Cavity

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Introduction

IT is generally recognized that the problem of separation is very important in practical aerodynamics and in many other fluid flowfields. Although a great deal of theoretical work has been done on steady separation, mostly through the use of the boundary-layer equations, the unsteady situation has received little attention in spite of its significance. Because the boundary-layer approximation is incomplete, recourse has been made to the full momentum equation. A two-dimensional oscillatory cavity flow presents a tractable model for approximate finite-difference solution of the equation to study the unsteady separation process.

Although other workers have used steady cavity flow models, this is the first time an oscillatory cavity flow solution has been published. The results are first compared to the steady case where there is only one fixed separation streamline along the line of symmetry. Then a dynamic pattern of separation and reattachment is illustrated.

Oscillatory Cavity Flow

Steady cavity flow induced by wall motion has drawn the attention of many workers[1] and the symmetric flow configuration, Fig. 1, was considered earlier, analytically, by Weiss and Florsheim.[2] The plane of mirror symmetry is represented by a straight dividing streamline, separating two counter-rotating vortices (one shown). More complete numerical solutions of the steady momentum equation at higher Reynolds numbers show that the vortex center locations are modified as the Reynolds number increases. However, the symmetry separation streamline is common to all the steady solutions.

Unsteady oscillatory flow can be induced in the cavity, driven by top and bottom plates moving in unison with a harmonic function; $U = U_m \cos \omega t$ (where U is horizontal velocity, U_m the

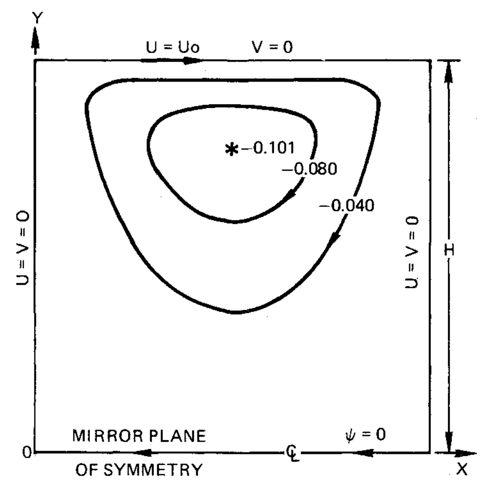

Fig. 1 Streamlines for a steady symmetrical Stokes cavity flow with viscous boundary conditions indicated (ψ is Stokes stream function; U, V are x, y velocity components; * marks a vortex center; upper half only shown).

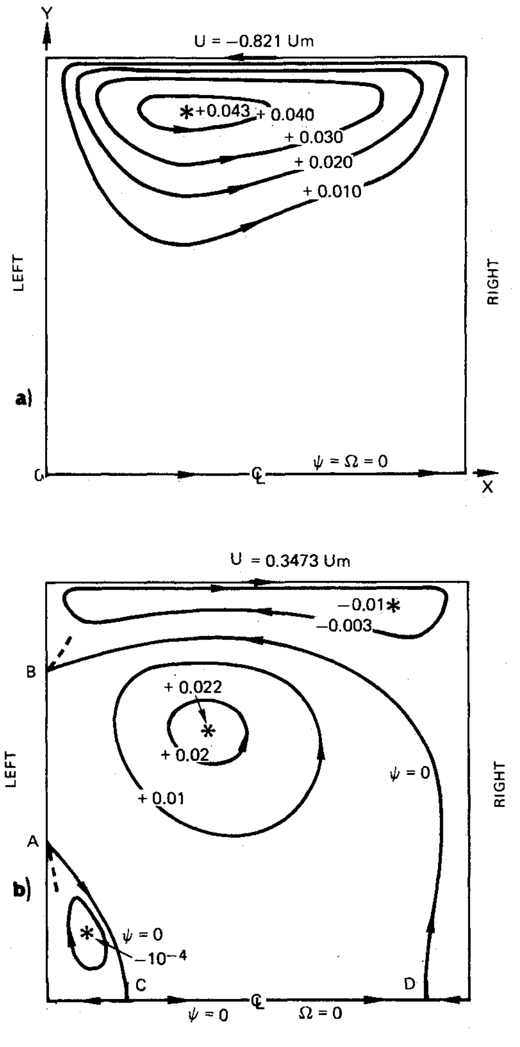

Fig. 2 Instantaneous streamlines for oscillatory symmetrical cavity flow ($U = U_m \cos \omega t$): $\hat{Re} = 100$; $N = 85$ (upper half). a) $\omega t = 2.534$ and b) $\omega t = 5.067$.

absolute magnitude of the oscillation and ω the frequency). Closed form unsteady analytic solutions are possible if the flow is parallel; but such solutions are not appropriate for the geometry illustrated in Fig. 1. Numerical finite-difference approximate solutions can be developed (after Ref. 3) and they are a function of two dynamic parameters, peak Reynolds number, $\hat{Re} = U_m H/\nu$ (where H is cavity half-height and ν is fluid kinematic viscosity) and a reduced frequency (Stokes number) $N = \omega H^2/(2\nu)$. The vortex behavior and the dividing streamlines that separate the vortices are now periodic functions of time.

Streamlines for an example ($\hat{Re} = 100$; $N = 85$) are shown at two instants during a cycle in Figs. 2a and b. Qualitatively, when there is only one vortex in each half of the cavity, Fig. 2a, the streamline pattern resembles the steady cavity flow at the same (instantaneous) Reynolds number, $Re = UH/\nu$. There is only a resemblance; the vortex center location and the magnitude of the flux extremum at that point are different from the steady case. The vortex center(s) tends to be closer to the top (bottom) and closer to the sides than the corresponding steady cavity flow. There is only the symmetry separation streamline.

A considerable difference from the steady flow is illustrated in Fig. 2b. There are two "extraneous" vortices at this instant in the cycle. They are due to residual motion from the past half-cycle of oscillation. When the direction of the moving walls reverses, the newly induced vortices have to compete with the old ones. This snapshot of the flow, Fig. 2b, shows *at the same time* three types of dividing streamline intersections: detachment from a wall at A, reattachment to a wall at B, and free intersection of streamlines at C and D.

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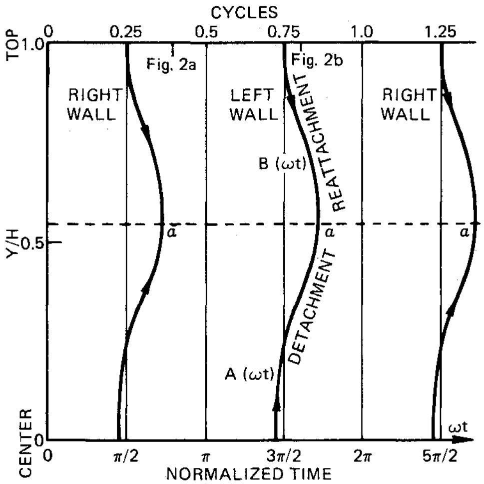

Fig. 3 Location of detachment and reattachment points on the vertical walls of the upper half of the cavity. (a—marks the time of annihilation of the "old" vortex.)

Look closely at the intersection regions near C and D. Note the angles; the intersecting streamlines are orthogonal, a fact noted previously for steady streamlines from a (local) Stokes analysis.[4] The points C and D are true stagnation points even though they occur out in the flow stream. The velocity gradients are also zero at the same location. The contour of zero vorticity magnitude Ω (or $|\text{curl}\ \mathbf{q}|$ where $\mathbf{q}$ is the normalized velocity vector) coincides with a streamline at the internal stagnation points. This is related to a conjecture of N. Rott, W. R. Sears, and F. K. Moore that midstream separation can be characterized by the coincidence of the velocity and shear zeroes. However, some of the hypothetical sketches[5] accompanying their conjecture suit a trifurcation of flow although a four-way partition always occurs in the present cavity calculation. Trifurcation is different.

Consider the vicinity of the detaching streamline ($\psi = 0$) at A on the wall; the $\Omega = 0$ contour (dashed) also goes through the intersection point. At the wall Ω can be considered a normalized shearing stress (skin friction). Here the angles of the streamline and the vorticity contour are neither coincident nor orthogonal to the wall. However, the angles bear a definite relationship to each other. If the angle of the vorticity contour is $\phi = \arctan \beta$, then the angle of the detaching streamline is $\gamma = \arctan (3\beta)$. Also the contour $V = 0$ intersects the vertical wall at the same point and it has angle $\eta = \arctan (2\beta)$. These relations are algebraic identities and cannot really be deduced from the nonlinear flow approximations. However, they have been demonstrated for steady flow separation[6,7] and the numerical results support the same relationships for the unsteady flows. The maximum pressure gradient is parallel to the zero vorticity contour.[6]

Similarly at the reattachment point B on the wall, the dividing streamline $\psi = 0$, the vorticity contour $\Omega = 0$ and the null vertical velocity contour all intersect. The angular relationships are the same as above, just the sign of the velocity vectors are reversed near the stagnation point.

As the harmonic cycle proceeds after the instant of Fig. 2b, the new vortices get bigger and stronger, while the other vortices get squeezed and diminished in velocity. Although the numerical program provides information to plot streamline and vorticity contours (or velocity components) for the cavity fields for every time-step in the computer program, it is sufficient to follow time-dependent A and B along the left wall to infer the separation pattern changes. As time progresses (Fig. 3), $B \to A$ until they merge and the attached vortex vanishes.

During the next half-cycle similar events occur on the opposite vertical wall. Once the driving plate changes direction, the newly induced tangential shear (of opposite sign to the previous half-cycle) forces the closure of the attached vortex to the side wall. As the "younger" vortex grows, its interaction with the residual motion forces the attachment and separation points to change. Eventually the older attached vortex disappears. And so it goes on in the oscillatory flow, vortex birth and death, cycle after cycle.

Conclusion

Many of the current concepts of unsteady flow separation have been developed from calculations using the (incomplete) boundary-layer equations.[8,9] Contrary to these "singularity" analyses, our numerical approximate solutions to the full momentum equation show separation is regular when transverse pressure gradients are included in the theory. So the use of the full equation permits a clearer view of the physical process. Coincidence of vorticity and velocity zeroes, on the wall or in the stream, marks a stagnation point and dividing streamline intersection, in steady or unsteady incompressible flow.

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